MATH 306 Workshop

1. Define what it means for a square matrix A be **invertible**.

2. Conditions equivalent to diagonalizability. (Thm 5.41) **Important**

   a. T is diagonalizable.

   b.

   c.

   d.

   e.

3. Let $T \in L(\mathbf{F}^5)$ and given an eigenvalue $\lambda = 3$ with corresponding eigenvectors v_1, v_2, v_3, v_4. Prove that $\lambda = 2$ and $\lambda = 5$ cannot both be eigenvalues of T.

4. Let $T \in L(\mathbf{V})$, and B_1, B_2 be bases for **V**. Explain the notation: $\mathbf{M(T, B_1, B_2)}$.

5. Let B be the standard basis for $\mathbf{R}^3$. Let B' be (0, 1, 0), (1, 0, 1), (1, 0, -1).

   Find M (I, B, B')

6. Let T be the linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ given by

   $$T(x, y, z) = (3x, 6z - x, -x + 3z)$$

   Let B be the basis {(1, 0, 1), (1, 0, −1), (0, 1, 0)}. Find M (T, B, B).

7. Let T be the linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ given by

   $$T(x, y, z) = (3x, 6z - x, -x + 3z)$$

   Let B be the standard basis. Let B’ be the basis {(0, 0, 1), (0, 1, 0), (1, 0, 0)}.

   Find M (T, B, B’).

8. Suppose A and B are square matrices of the same size and AB = I. Prove that BA = I.

9. Let T and S be the linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ given by

   $T(x, y, z) = (3x, 6z - x, -x + 3z)$

   $S(x, y, z) = (6z, 3y - z, x + y)$

   Let B be the standard basis. Let B' be the basis {(1, 0, 1), (1, 0, −1), (0, 1, 0)} and B'' be the basis {(0, 0, 1), (0, 1, 0), (1, 0, 0)}.

   (a) Find M (T, B, B'), M (S, B', B''), and M (ST, B, B'').

   (b) Calculate M (S, B', B'') * M (T, B, B'). Then compare the result with M (ST, B, B'')